Trigonometric problems in two and three dimensions.

96. A mast, height 43 m, is secured by two wire ropes from points A and C.

The distance from A to C is 76 m.

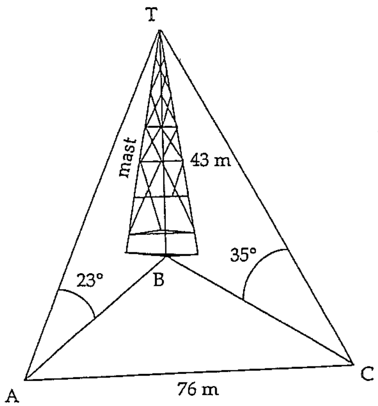
The top of the mast has angles of elevations of 23° and 35° from A and C. A, B and C are all on level ground.

Find

a) the distance AB.

b) the angle ∠ABC.

c) the angle between the ropes at the top of the mast (∠ATC).



97. A student walking towards a flag pole notes that the angle of elevation to the top of the pole changes from x to 2x over a distance of a m.
Find the height h of the flag pole in terms of a and angle x.

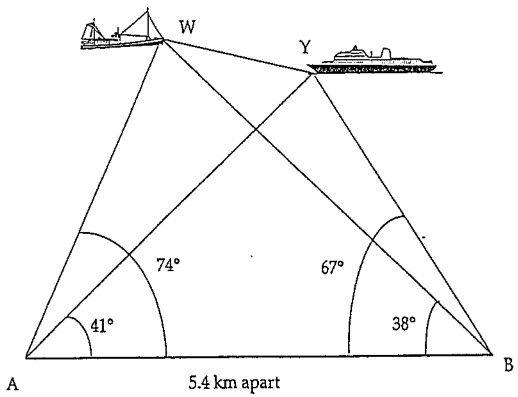
Hint. On your diagram mark in as many angles as you can.

98. Two coast guard stations exactly 5.4 km apart measure the angles to two ships as shown in the diagram.

Find the distance between the two ships W and Y.

Hint. You will need the sine rule and the cosine rule.

Note. The question is stated in degrees.



99. Two rugby players are on opposite sides of the goal posts at one end of a rugby field. They are 15 metres apart. The angles of elevation to the cross bar are 58° and 36° respectively.

a) How high is the cross bar?

b) If their distance apart is represented by the letter x and the angles of elevation are a and b respectively, derive a formula for the height h of the cross bar.

100. A jogger running along a level road towards a tower calculates at one point the angle of elevation to the tower to be 15°. 150 metres further along the road the angle of elevation is calculated to be 45°. Calculate the height of the tower.

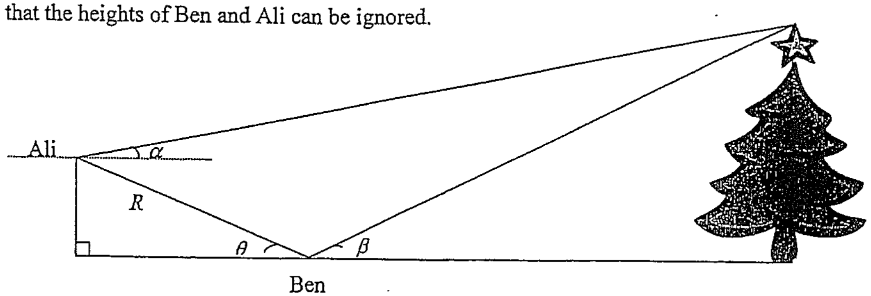
Question 9 *(Answer p184)*

Ali and Ben are skateboarding in the park where there is a brightly decorated Christmas tree.

Ali, poised at the top of a ramp, looks at the top of the tree at an angle of elevation of α. Ben, at the bottom of the ramp, also looks at the top of the tree, but at an angle of elevation of β.
Ben, Ali, and the tree are in a straight line, the ramp slopes at an angle of θ to the horizontal, and the distance down the ramp between Ali and Ben is R metres, as shown on the diagram below.

Show that the height, h, of the tree, is given by $h = \dfrac{R\tan\beta\left(\sin\theta + \cos\theta\tan\alpha\right)}{\tan\beta - \tan\alpha}$

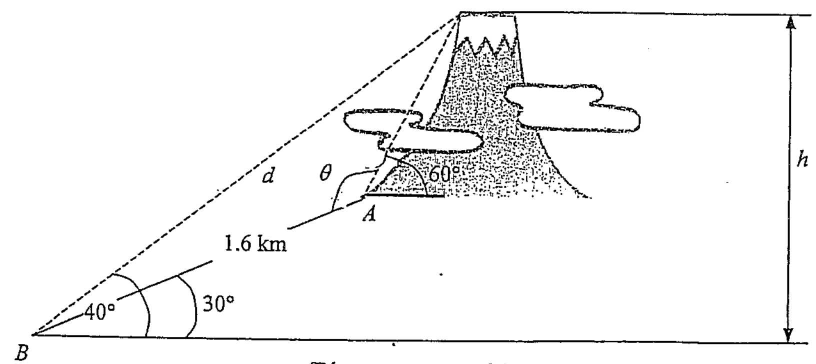
Assume that the heights of Ben and Ali can be ignored.



(4)

Question 20 *(Answer p193)*

A climbing party takes bearings to the top of a mountain. From their base camp B at the end of the road, the angle of elevation is measured at 40°. From their advance camp at A, the angle of elevation is 60°. The advanced camp is 1.6 km closer to the mountain along a straight ridge, which rises directly towards the top of the mountain at a steady angle of 30° from base camp.



(Diagram not to scale)

(a) Determine the distance marked d.

(b) Calculate the height h of the top of the mountain above base camp level. (5)

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96. a) AB $= 101$ m

b) Use the cosine rule

$\angle ABC = 48.4°$

c) $\angle ATC = 43.6°$

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97. look for an isosceles triangle

$h = a \sin 2x$

98. AY $= 5.23$

AW $= 3.59$

WY $= \sqrt{3.59^2 + 5.23^2 - 2{\times}3.59{\times}5.23 \cos 33°}$

$= 3.0$ km (2 sf)

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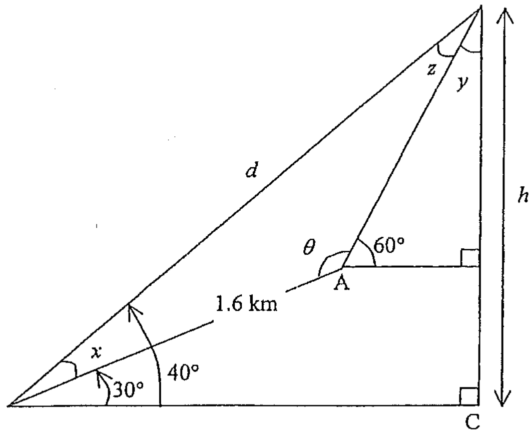
99. a) cross bar $= 7.5$ m (2 sf)

b) $h = \dfrac{x}{\cot a + \cot b}$

or $h = \dfrac{x \tan a \tan b}{\tan a + \tan b}$

100. $h = 54.9$ m (3 sf)

Question 20 *(Question p71)*



(a) $x = 10°$

Since the internal angles of a triangle add to 180°,

$y = 30°$

$y + z = 50°$

$z = 20°$

and $\theta = 180° - 20° - 10°$

$= 150°$

Now by the sine rule,

$$\frac{1.6}{\sin 20°} = \frac{d}{\sin 150°}$$

$$d = \frac{1.6 \times \sin 150°}{\sin 20°}$$

$= 2.3390$ km

$= 2.34$ km (3 sf)

(b) $\sin 40° = \dfrac{h}{d}$

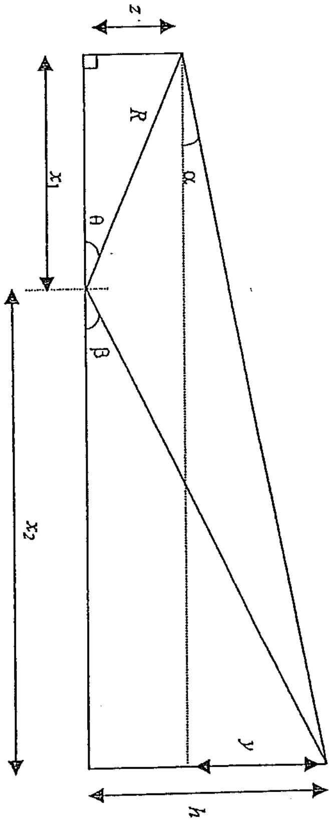
$= \dfrac{h}{2.3390}$

$h = 2.3390 \sin 40°$

$= 1.5034$ km

$= 1.50$ km (3 sf)

Question 9 *(Question p67)*



$z = R\sin\theta;$

$x_1 = R\cos\theta \quad y = (x_1 + x_2)\tan\alpha$

$\tan\beta = \dfrac{h}{x_2}$ and $y = \dfrac{h}{\tan\beta}$

$h = z + y$

$$= R\sin\theta + \tan\alpha\left(R\cos\theta + \frac{h}{\tan\beta}\right)$$

$$= R\sin\theta + R\tan\alpha\cos\theta + h\frac{\tan\alpha}{\tan\beta}$$

$$h\left(1 - \frac{\tan\alpha}{\tan\beta}\right) = R\sin\theta + R\tan\alpha\cos\theta$$

$$h = \frac{R\sin\theta + R\tan\alpha\cos\theta}{\left(1 - \dfrac{\tan\alpha}{\tan\beta}\right)}$$

$$= R\frac{(\sin\theta + \tan\alpha\cos\theta)}{\left(\dfrac{\tan\beta - \tan\alpha}{\tan\beta}\right)}$$

$$= \frac{R\tan\beta(\sin\theta + \cos\theta\tan\alpha)}{\tan\beta - \tan\alpha}$$